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Self-similar optical transmittance for a deterministic aperiodic multilayer structure

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Abstract

We study theoretically the spectral transmission properties of a multilayer structure in which the refractive index of the layers follows a self-similar arithmetical sequence named 'The 1s-counting sequence', which is related to the Pascal's triangle. The transmittance spectrum is intermediate between that of a periodic structure and that of a random structure, and shows clearly properties of scaling and self-similarity for all incident angles and TE and TM polarizations.

1. Introduction

Since the experimental work on quasicrystals by Shechtman *et al* [1], intensive investigations have been made of the structure and physical properties of quasiperiodic deterministic structures such as one-dimensional quasiperiodic multilayers. These systems present properties not observed in periodic or completely disordered structures. Merlin *et al* [2] were the first to grow a Fibonacci lattice of GaAs and AlAs and they studied its x-ray diffraction and Raman scattering properties. Since then, many other interesting studies of propagation of electrons, electromagnetic waves and acoustic waves have been reported in quasiperiodic multi-layered one-dimensional structures with several profiles like those of Fibonacci, Cantor, Rudin–Shapiro, Thue–Morse, and others [3–5]. Of particular interest is the understanding of the optical propagation, localization and transmission of optical waves in this type of structures, which are different from the periodic or random structures due to the long correlation effects induced by the quasiperiodicity [6]. These multilayers may have promising technological applications in non-linear optics as well as in the design of optical devices like soft x-ray filters, efficient photovoltaic solar cells [6–8], etc. In this work we propose a model for a new aperiodic multilayer whose refractive index is modulated by a deterministic numerical sequence named 'the 1s-counting sequence', formed by the number of 1s in the binary representation of the natural numbers [9]. This sequence is related to the self-similar sequence formed by the quantity of odd entries in each row of the Pascal's triangle, which is named the Gould's sequence [9]. In an article published formerly [10] a calculation was made of the electronic transmission for a finite superlattice where the barrier width is modulated by the Gould's sequence. For the calculations we use the transfer matrix method, where the transfer matrix is formed by the product of dynamical matrices for each interface and propagation matrices for each layer.

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2. Theoretical model and method of calculation

The 1s-counting sequence is generated by counting the number of 1s in the binary representation of the natural numbers. For example, the natural numbers 0, 1, 2, 3, 4, 5, 6, 7, etc can be represented, respectively, in the binary numeral system by 0, 1, 10, 11, 100, 101, 110, 111, etc. Counting the number of 1s in each binary representation, we are left with the sequence $P = 0, 1, 1, 2, 1, 2, 2, 3, \ldots$. This sequence can be constructed recursively. For step zero, we take 0 as the initiator. For step one, we sum 1 to 0 and we make a concatenation with the initiator, obtaining 0, 1. For step two, we again sum 1 to these last numbers and we make again a concatenation with the preceding numbers to obtain 0, 1, 1, 2. For step three, we sum 1 once more to the four last numbers and we make again the concatenation to obtain 0, 1, 1, 2, 1, 2, 2, 3, and so on. Every step doubles the length of the sequence. Then, we construct the sequence following the recursive formula $A_j = \{A_{j-1}\}$ & $\{1 + A_{j-1}\}$; for $j = 0, 1, 2, \ldots, \infty$, with $A_0 = 0$ as the initial value; here the symbol & means concatenation. For step S, the number of terms of the sequence is 2^S. Since the sequence can be constructed recursively, it has the property of self-similarity. For example, if we take the sequence by pairs (0, 1), (1, 2), (1, 2), (2, 3), ..., this sequence also follows the original sequence, and also for quartets, octets, etc. Also, if we underline every second term of the sequence, we can reproduce the original sequence.

The multilayer structure we propose is made of $N = 2^L$ layers, each one having a refractive index given by

$$n_P = n_1 + (\Delta)(P) \tag{1}$$

where n_1 is an initial refractive index for the first layer, Δ is a small increment, $S = L$ is the number of the step for the sequence, and P takes the values of the terms of the sequence for a given value of S, for each layer of the structure, whose growth direction is the x axis. This multilayer structure can be easily fabricated using the available technology as has been recently demonstrated for the case of a Fibonacci one-dimensional multilayer [11] prepared using the porous silicon technology.

In order to calculate the transmittance, we use the theory presented in [12] to build the transfer matrix of the structure. Considering the xz plane as the plane of incidence, the electric field vector of a plane wave, which is a solution of the electromagnetic wave equation, can be written as

$$\vec{E}(\vec{r}, t) = \vec{E}(x)e^{i(\beta z - \omega t)}$$

for an isotropic and homogeneous medium in the z direction. Here β is the z component of the propagating wavevector, and ω the angular frequency. The magnetic field is given by $\vec{B} = \frac{1}{i\omega}\nabla \times \vec{E}$. The electric field in the incident medium (refractive index n_0), in the layers, and in the substrate (refractive index n_s) can be written as

$$E(x) = E_{1j}e^{ik_j x} + E_{2j}e^{-ik_j x}.$$

Here the first and second terms represent waves propagating to the right and to the left in the x direction, and $k_j = (\omega/c)n_j \cos\theta_j$ is the wavevector for medium j, with $j = 0, 1, \ldots, N, S$; being N the number of layers in the structure. Considering only propagation to the right, $E_{2s} = 0$. Propagation from medium 0 to medium S through the multilayer structure can be described by

$$\begin{pmatrix} E_{10} \\ E_{20} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} E_{1S} \\ 0 \end{pmatrix}$$

where the 2×2 transfer matrix is given by

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = D_0^{-1}\left(\prod_{l=1}^{N} D_l P_l D_l^{-1}\right) D_S$$

with

$$P_j = \begin{pmatrix} e^{-k_j d_j} & 0 \\ 0 & e^{ik_j d_j} \end{pmatrix}.$$

Here

$$D_j = \begin{pmatrix} 1 & 1 \\ n_j \cos\theta_j & -n_j \cos\theta_j \end{pmatrix} \qquad \text{for TE polarization}$$

and

$$D_j = \begin{pmatrix} \cos\theta_j & \cos\theta_j \\ n_j & -n_j \end{pmatrix} \qquad \text{for TM polarization.}$$

The matrix D_j is called the dynamical or transmission matrix, and arises from the continuity conditions on the electric and magnetic fields at the interface between media $j-1$ and j. P_j is the kinematical or propagation matrix inside layer j, with d_j the thickness of the layer. The transmittance T is given by the ratio of the Poynting power flow of the transmitted wave to that of the incident wave, and is given in terms of the transfer matrix, by

$$T = \frac{n_s \cos\theta_s}{n_0 \cos\theta_0}\left|\frac{1}{M_{11}}\right|^2.$$

We suppose nondispersive media without absorption.

3. Numerical results

We consider that the multilayer structure is situated in vacuum, with $n_0 = n_S = 1.0$. We take $n_1 = 2.0$ and $\Delta = 0.1$ in (1). The optical thickness $n_j d_j$ of the layers is a quarter wavelength $\lambda_0/4$, where λ_0 is a central wavelength in the vacuum. In figure 1 we show the refractive index profile of three finite multilayer structures; figure 1(a) is constructed with 32 equal unit cells, each one having eight layers with refractive index 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, and 2.8 in that order, which can represent a periodic structure with a well defined period. The deterministic aperiodic structure in figure 1(b) is formed by generating the refractive indices n_p using (1), where the refractive index is modulated by the 1s-counting sequence; here we cannot define a specific unit cell as is the case for aperiodic structures. Finally, the disordered structure in figure 1(c) is constructed using the same previous values for n_P, but randomly assigned to each layer; here also we cannot define a specific unit cell. In all three cases the structure has $N = 256$ layers. We can construct multilayer stacks having $N = 2^L$ slabs with $L = 1, 2, \ldots, 7$, finding similar results, as we explain later.

The transmittance for these three multilayers is shown in figure 2 for normal incidence. In figure 2(a) the spectrum for the periodic structure shows four gaps where the transmittance $T = 0$. Figure 2(b) corresponds to the deterministic aperiodic multilayer. Here the minima of T are located in the same positions as in figure 2(a), and besides this new minima appear in the transmittance T. The spectrum for the random structure shown in figure 2(c) has no gaps, as expected. We observe that for long wavelengths compared to the wavelength in vacuum, the transmittance is very similar for the three structures, whereas for wavelengths comparable with the wavelength in vacuum the periodic structure shows Bragg

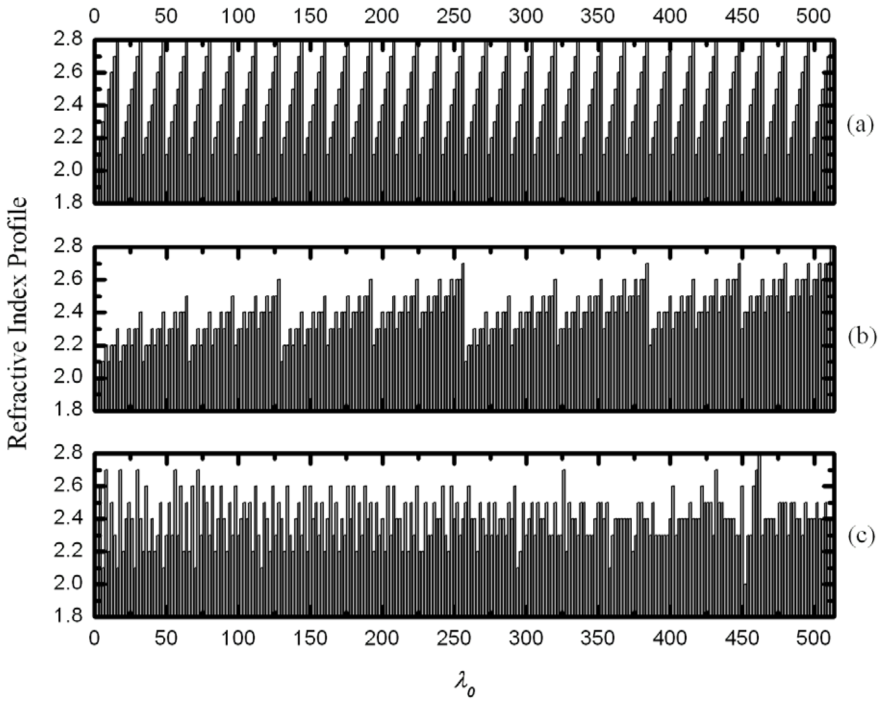

Figure 1. Refractive index profile as a function of the optical thickness for structures with $N = 256$ layers in air with $n_0 = n_S = 1$: (a) periodic structure formed with 32 cells, each one having eight slabs with refractive index values 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8; (b) deterministic aperiodic structure with refractive index $n_P = n_1 + (\Delta)(P)$ modulated by the 1s-counting sequence; here $n_1 = 2.0$ and $\Delta = 0.1$; (c) random structure formed with the same refractive index as in (b) now accommodated at random in the multilayer.

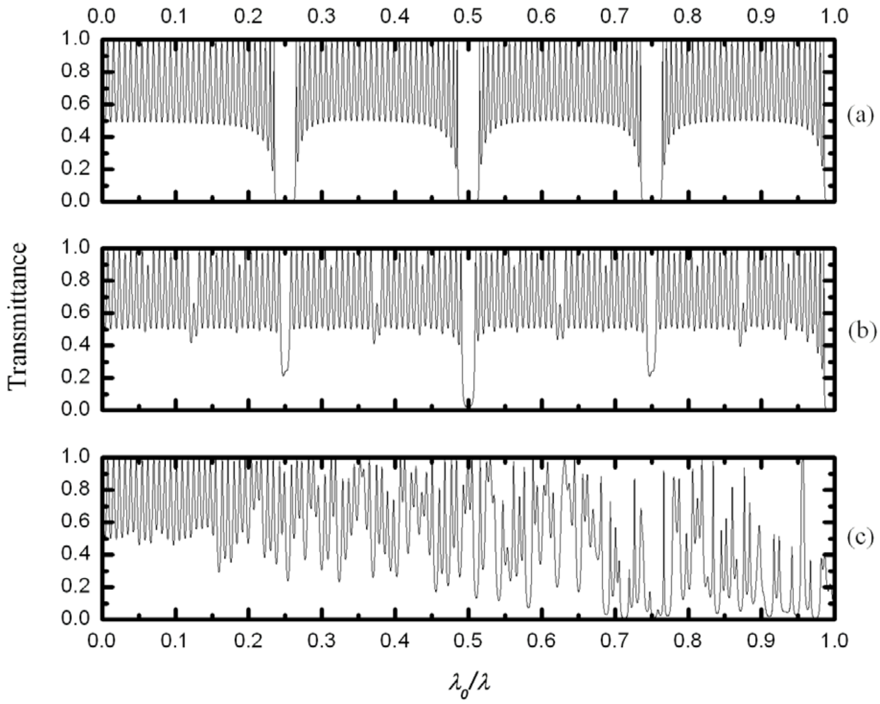

Figure 2. Transmittance spectrum for a structure with $N = 256$ layers as in figure 1. (a) Periodic, (b) deterministic aperiodic, (c) random.

transmittance whereas the transmittance for the aperiodic and random structures is self-similar or completely random. We see that the behavior of the transmittance spectrum for the deterministic aperiodic structure is intermediate between that of the periodic structure and the one for the random structure.

In figures 3–9 we compare the transmittances at normal incidence for the structures having $N = 2, 4, 8, 16, 32, 64$, and 128 slabs with the transmittance corresponding to a multilayer with $N = 256$ slabs. In figures 3–8 the dashed line corresponds to the transmittance versus λ_0/λ in the range 0.0–1.0 for the

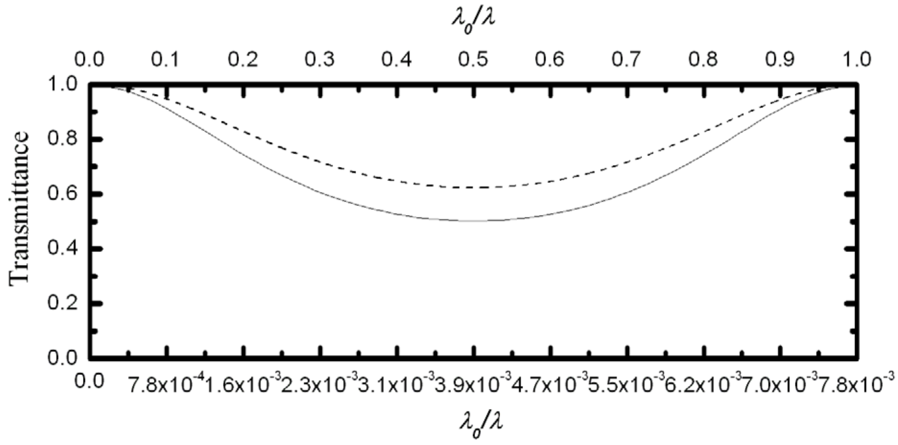

Figure 3. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The dashed line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 2$ slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 7$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs.

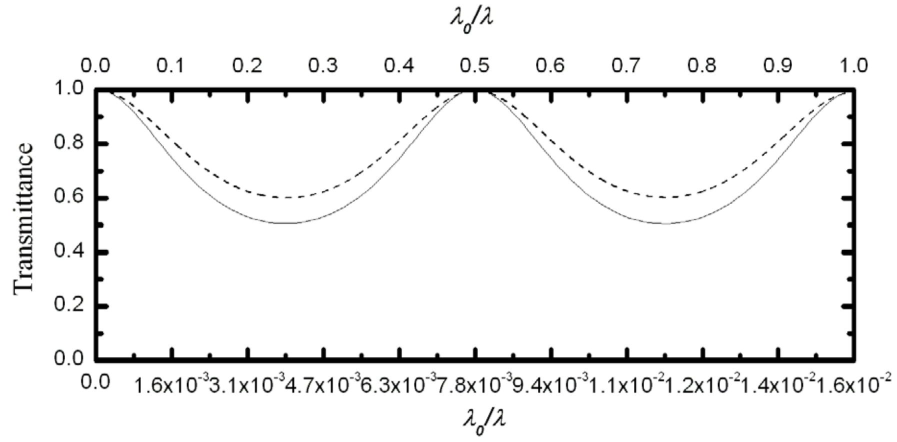

Figure 4. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The dashed line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 4$ slabs, the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 6$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs.

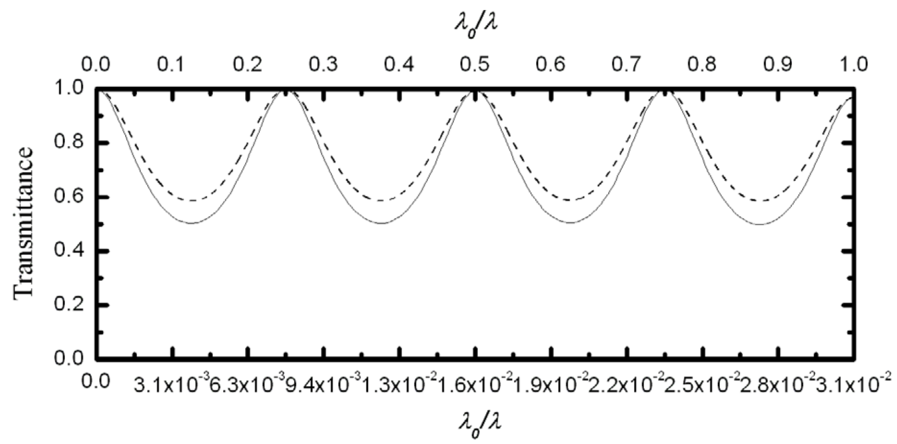

Figure 5. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The dashed line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 8$ slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 5$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs.

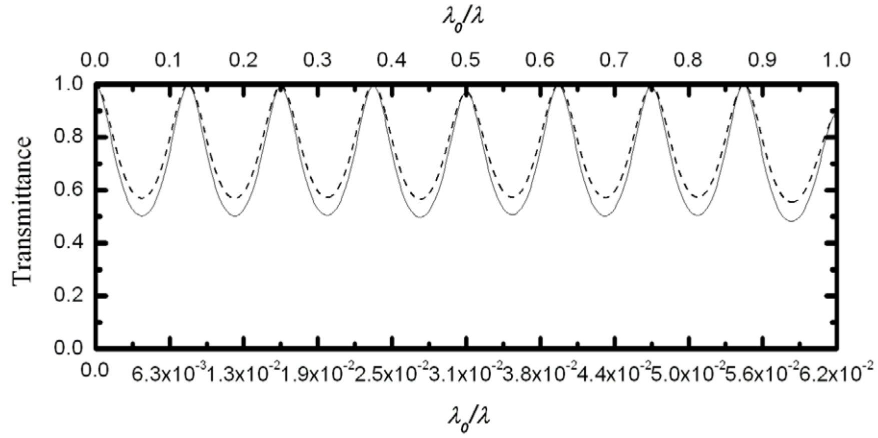

Figure 6. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The dashed line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 16$ slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 4$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs.

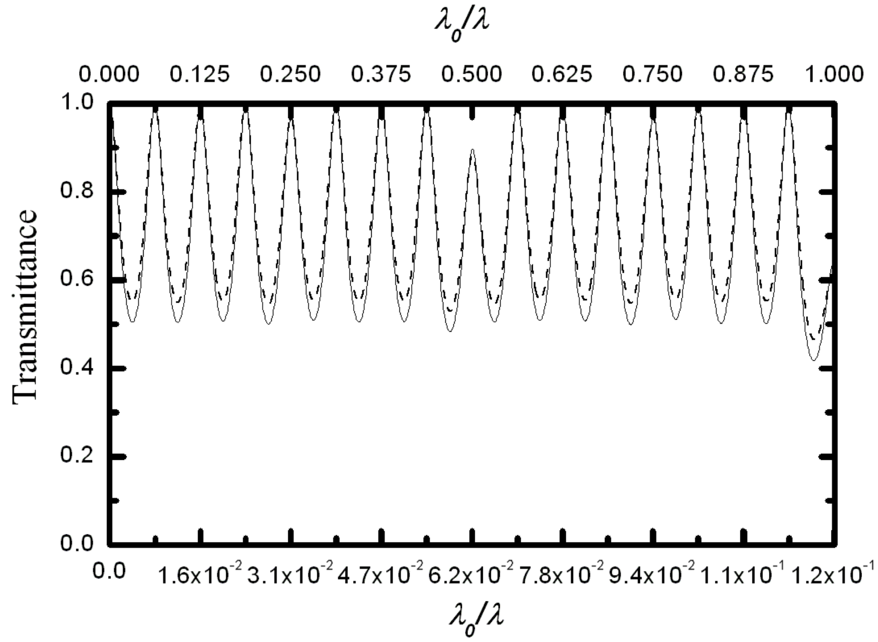

Figure 7. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The dashed line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 32$ slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 3$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs.

structures with $N = 2, 4, 8, 16, 32$, and 64 slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ ($L = 6, 5, 4, 3, 2, 1$) of the total range 0.0–1.0 for the superlattice with $N = 256$ layers. In figure 9, the circles represent the transmittance for a structure with $N = 128$ slabs in the range 0.0–1.0 of the parameter λ_0/λ and the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ ($L = 7$) of the total range 0.0–1.0 for the superlattice with $N = 256$ layers. In this case we observe that the transmittance spectrum is almost the same for the two structures shown.

We observe that the transmittance spectrum of the smaller structures is contained in the spectrum of the superlattice with $N = 256$ layers. Expanding the $1/2^L$ ($L = 1, 2, \ldots, 7$) part of the transmittance spectrum of the $N = 256$ layer stack in such a way that it coincides with the whole transmittance spectrum of the smaller multilayers, we see that the positions of the maxima and minima of the transmittance spectrum are the same for all the structures, the minima of the spectrum being deeper as the number N increases. Likewise, we find that the transmittance spectrum of the structure with $N = 128$ layers contains the transmittance of the multilayers with $N = 2, 4, 8, 16, 32$, and 64 slabs. Finally, we can say that the transmittance spectrum corresponding to a multilayer structure having $N = 2^L$ slabs contains the transmittance for the structures with $N = 2^{L'}$ ($L' = L - 1, L - 2, \ldots, 1$) in a

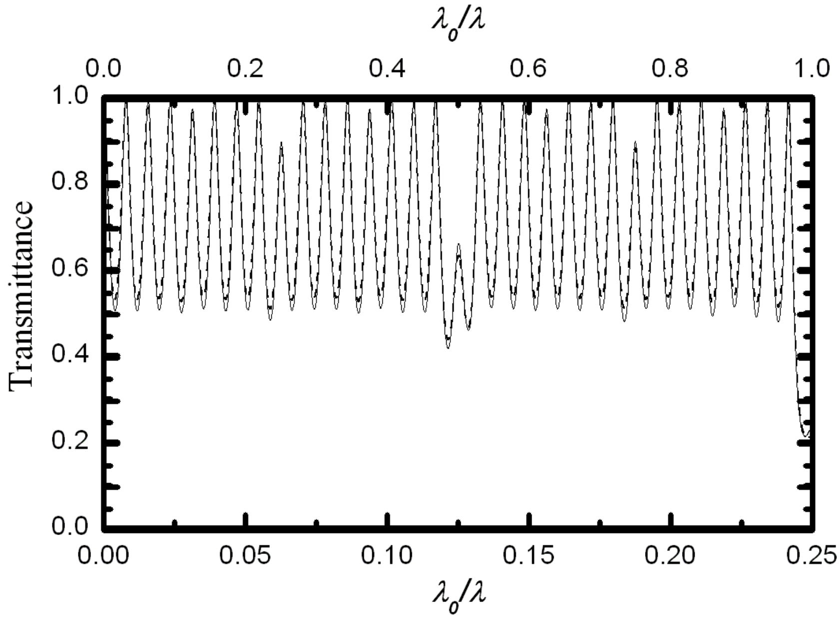

Figure 8. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The dashed line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 64$ slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 2$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs.

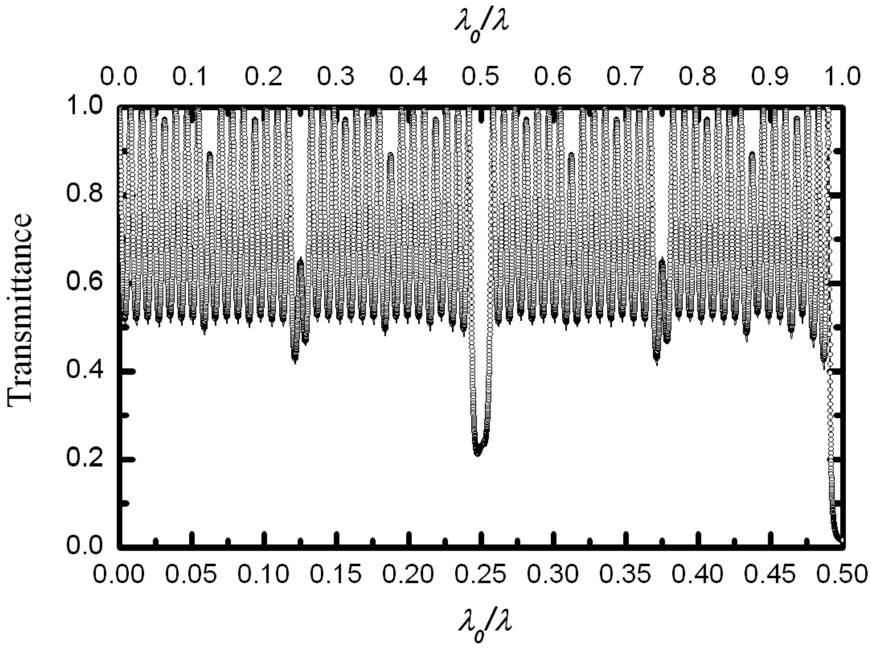

Figure 9. Transmittance spectra of multilayer deterministic aperiodic structures at normal incidence situated in vacuum. The circles corresponds to the transmittance as a function of λ_0/λ in the range 0.0–1.0 of a multilayer with $N = 128$ slabs; the solid line corresponds to the transmittance as a function of λ_0/λ in the range 0.0–$1/2^L$ with $L = 1$ of the total range 0.0–1.0 of the parameter λ_0/λ for the multilayer with $N = 256$ slabs. In this case both spectra are almost equal.

range equal to $1/2^{L''}$ ($L'' = 1, 2, \ldots, L-1$) the total range (0.0–1.0) of the parameter λ_0/λ for the larger structure. The scaling of the spectra by a factor of two is also a sign of self-similarity. This property is conserved for oblique incidence and both TE and TM polarizations. As an example, we calculate the transmittance at $\theta_0 = 45°$ represented in figures 10 and 11 with circles for the multilayer with $N = 64$ slabs, and compare it with the transmittance of the multilayer with $N = 256$ for the same polarizations represented by a solid line.

Here we see that the spectrum of the smaller structure is contained in the transmittance of the larger multilayer as in the case of normal incidence. This scaling of the transmittance spectra by a factor of two is expected from the self-similarity of the structure by the same factor. One structure of L steps

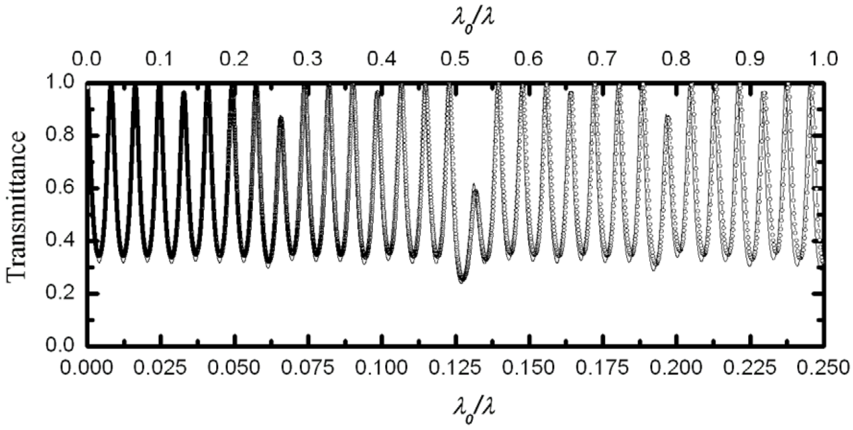

Figure 10. Transmittance spectra for incidence angle $\theta_0 = 45°$. For TE polarization we see that the spectrum of the structure with $N = 64$ layers shown as circles in the figure is contained in the spectrum for the multilayer with $N = 256$ layers. Here the parameter λ_0/λ goes from 0.0 to 1.0 for the structure with $N = 64$ layers, and from 0.0 to $1/2^2$ for the structure with $N = 256$ layers. In this case the transmittance is very similar for both structures.

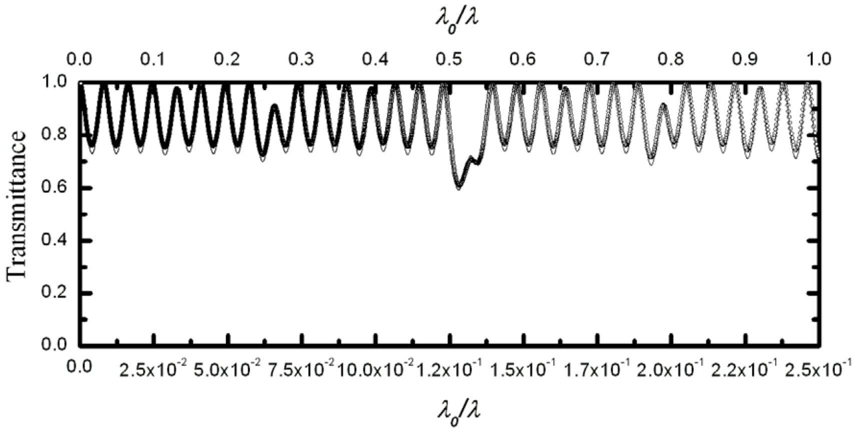

Figure 11. Transmittance spectra for incidence angle $\theta_0 = 45°$. For TM polarization we see that the spectrum of the structure with $N = 64$ layers shown as circles in the figure is contained in the spectrum for the multilayer with $N = 256$ layers. Here the parameter λ_0/λ goes from 0.0 to 1.0 for the structure with $N = 64$ layers, and from 0.0 to $1/2^2$ for the structure with $N = 256$ layers. In this case the transmittance is very similar for both structures. We observe a better transmittance for TM polarization.

encompasses the structures of all previous steps. Suppose, for example, that we have the structure of $N = 256$ layers and we take only the part corresponding to 128 layers; if we multiply by two the phase shift suffered by a plane wave in this part, we obtain the same phase shift as that of a plane wave that travels through the structure of 256 layers. This is equivalent to reducing the wavelength by half, or multiplying the frequency by two. The scaling is also observed for the transmittance spectra of Cantor-like multilayer structures [4], and it has been demonstrated analytically that spectral self-similarity is a characteristic property of geometrical quasiperiodicity [13]. For the structure presented in this work, observe self-similarity even in structures with very few layers. These results are valid for any value of λ_0 with λ_0/λ in the interval [0, 1].

4. Conclusions

In this work we present a theoretical analysis of the transmittance of a new deterministic aperiodic multilayer structure whose refractive index profile is modulated by the self-similar aperiodic 1s-counting sequence. For equal optical thicknesses of the structure's layers, every incidence angle and both polarizations TE and TM, the transmittance spectrum for the deterministic aperiodic structure presented here shows self-similarity even with structures having only a few layers, the spectrum for these structures being intermediate between that produced by a periodic multilayer structure and the one produced by a disordered one. We find this behavior for every value of λ_0 with λ_0/λ in the interval [0, 1].

Acknowledgments

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